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Analysis of Differential Line Length Diplexers and Long-Stub Filters

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LINCOLN LABORATORY

ANALYSIS OF DIFFERENTIAL LINE LENGTH DIPLEXERS
AND LONG-STUB FILTERS

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Group 61

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ABSTRACT

Formulas for calculating the appropriate waveguide width for perfect performance at two center frequencies have been derived for the differential length diplexer and long-stub filter. The general characteristics of these devices are calculated in terms of phase angle with dissipation loss included. An anlysis of bandwidth, dimensional and thermal expansion tolerances for these devices is also given. A single algebraic equation relating the diplexer performance and the various parameter first-order deviations is obtained. This relation gives guide lines for adjusting final performance and for predicting the combined effects of dimensional, thermal and frequency deviations upon the general performances. Examples of diplexer and long-stub filter designs and analyses are given.

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Analysis of Differential Line Length Diplexers and Long-Stub Filters

I. INTRODUCTION

An ideal diplexer is a three-port device where port 1, say, accepts sig-
nal 1 with center frequency f_1, port 2 accepts signal 2 with center frequency
f_2; both signals emerge out of port 3 with zero loss and ports 1 and 2 are iso-
lated. Early diplexers used transmission line T or Y junctions with filters.[1]
The required isolation is accomplished by filter rejection and a good junction
match is accomplished by suitably positioning each filter with respect to the
junction. The basic disadvantage of using T or Y configurations in diplexer
design is that the filters must be located close to the common junction in
terms of the operating wavelength. In addition, in high frequency ranges these
devices are usually hard to tune.

In contrast, a differential line length diplexer using two hybrid junctions
is a four-port device; it is inherently well matched and allows each filter to be
located at virtually any distance from the common junction. For a given loss-
less waveguide, the theory of this device is well known and a graphical design
procedure is available in the literature.[2]

For a given waveguide size, such as standard waveguide, in general, there
is no suitable differential length that will give perfect performance at the two
given center frequencies f_1 and f_2. To provide a greater flexibility of diplexer
design, the waveguide width of the section of differential length can be varied.
In the present work, an expression for the appropriate waveguide size, which

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allows for perfect performance at two given center frequencies, is obtained.

The dissipation loss for coaxial lines at frequencies up to X-band and for small waveguides at frequencies above X-band is not always negligible. Moreover, in the EHF range, the waveguide sizes are so small that dimensional tolerance, thermal expansion and frequency deviations may become quite important. It is for these purposes that the diplexer performance versus dissipation loss, frequency deviation, dimensional tolerance, and thermal expansion detuning is analyzed.

The isolation between ports 1 or 2 of the diplexer is determined by the isolation between the sum and difference ports of the hybrid junctions, the leakage signal from port 3 to ports 1 and 2, and mismatch from the terminals. In many applications, this isolation is not sufficient. Additional isolation can be accomplished by the addition of filters or by cascading similar differential line length diplexers.

Conventional filter designs generally lead to complex structures, such as irises and coupling holes, which require close dimensional tolerances. In the EHF range, the stringent tolerance requirements are critical and also extra dissipation loss may be introduced by the complex structures due to the presence of a large number of evanescant modes. These difficulties are minimized in the case of long-stub filters and differential line length diplexers because the performance of these devices depends mainly on the largest dimensions of the transmitting system, namely, waveguide width and length. Because of

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these interesting features, the long-stub filter is also analyzed here.

II.   GENERAL CHARACTERISTICS OF THE DIPLEXER

To facilitate explanation, a schematic drawing of the basic diplexer circuits is shown in Fig. 1. The diplexer consists of two hybrid junctions whose side arms are connected by transmission lines of different lengths. If the difference in length, L, is adjusted so that input signals at f_1 (say the transmitter frequency) that are incident at port 3 arrive at the output hybrid junction in phase. Whereas signals at f_2 (say the receiver frequency) that are incident at port 3 arrive 180° out-of-phase; the signals will proceed from the common port 3 (say antenna) to ports 1 and 2, respectively with minimum insertion loss. Port 1 (transmitter) and port 2 (receiver) are then isolated over a broad frequency band by an inherent property of the hybrid junction.

Consider the signal of unit power incident from the sum port of the antenna and assume that the two hybrids are matched and lossless. The power to the sum and difference ports of the other hybrid are given by

$$P_\Sigma = e^{-2\alpha L_1} \, e^{-\alpha L} \left(\cosh^2 \frac{\alpha L}{2} \cos^2 \frac{\theta}{2} + \sinh^2 \frac{\alpha L}{2} \sin^2 \frac{\theta}{2} \right) \tag{1}$$

$$P_\Delta = e^{-2\alpha L_1} \, e^{-\alpha L} \left(\sinh^2 \frac{\alpha L}{2} \cos^2 \frac{\theta}{2} + \cosh^2 \frac{\alpha L}{2} \sin^2 \frac{\theta}{2} \right) \tag{2}$$

where $L = L_2 - L_1$ is the difference in line length, $\theta = \beta L$, and α is the attenuation constant and $\beta = 2\pi/\lambda g$ is the phase constant of transmission lines between the two hybrid junctions. When the transmission line is lossless, $\alpha = 0$,
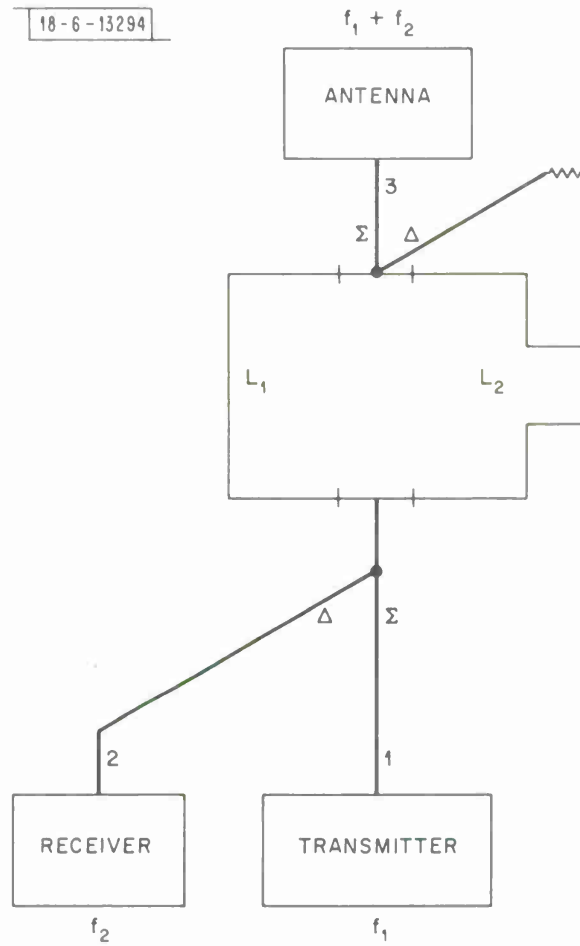
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Fig. 1.   A differential line length diplexer arrangement.

Eqs. (1) and (2) reduce to the familiar results $P_\Sigma = \cos^2 \frac{\theta}{2}$ and $P_\Delta = \sin^2 \frac{\theta}{2}$. For perfect performance, we need $\theta = 2m\pi$ at f_1 and $(2n-1)\pi$ at f_2 or

$$L = m\lambda g_1 = \frac{2n-1}{2} \lambda g_2 \tag{3}$$

where λg_1, λg_2 are the guide wavelengths at frequencies f_1 and f_2; m and n are integers. From Eqs. (1) and (2), the deviation from perfect performance can be expressed in terms of a change in θ with αL as a parameter, as shown in Fig. 2. It is seen that the diplexer performance deviation increases continuously with increasing θ and αL. For example, for $\alpha L = 0$, if $\Delta \theta / 2 = 8.5°$, the insertion loss is 0.1 dB and discrimination is about 17 dB. With finite loss, $\alpha L \neq 0$, the insertion loss and isolation are both finite even when Eq. (3) is satisfied.

For a standard waveguide size with two given center frequencies, in general there is no suitable differential length that will give perfect performance at both of the given frequencies. However, perfect performance can be obtained if the difference in length is made of waveguide with suitable size.

For unloaded waveguide, the dispersion relation is given by

$$\frac{1}{\lambda g^2} = \frac{1}{\lambda_o^2} - \frac{1}{\lambda_c^2} \tag{4}$$

where λ_o is the free-space wavelength and λ_c the cut-off wavelength. Substituting Eq. (4) into Eq. (3) and solving for λ_c, we have
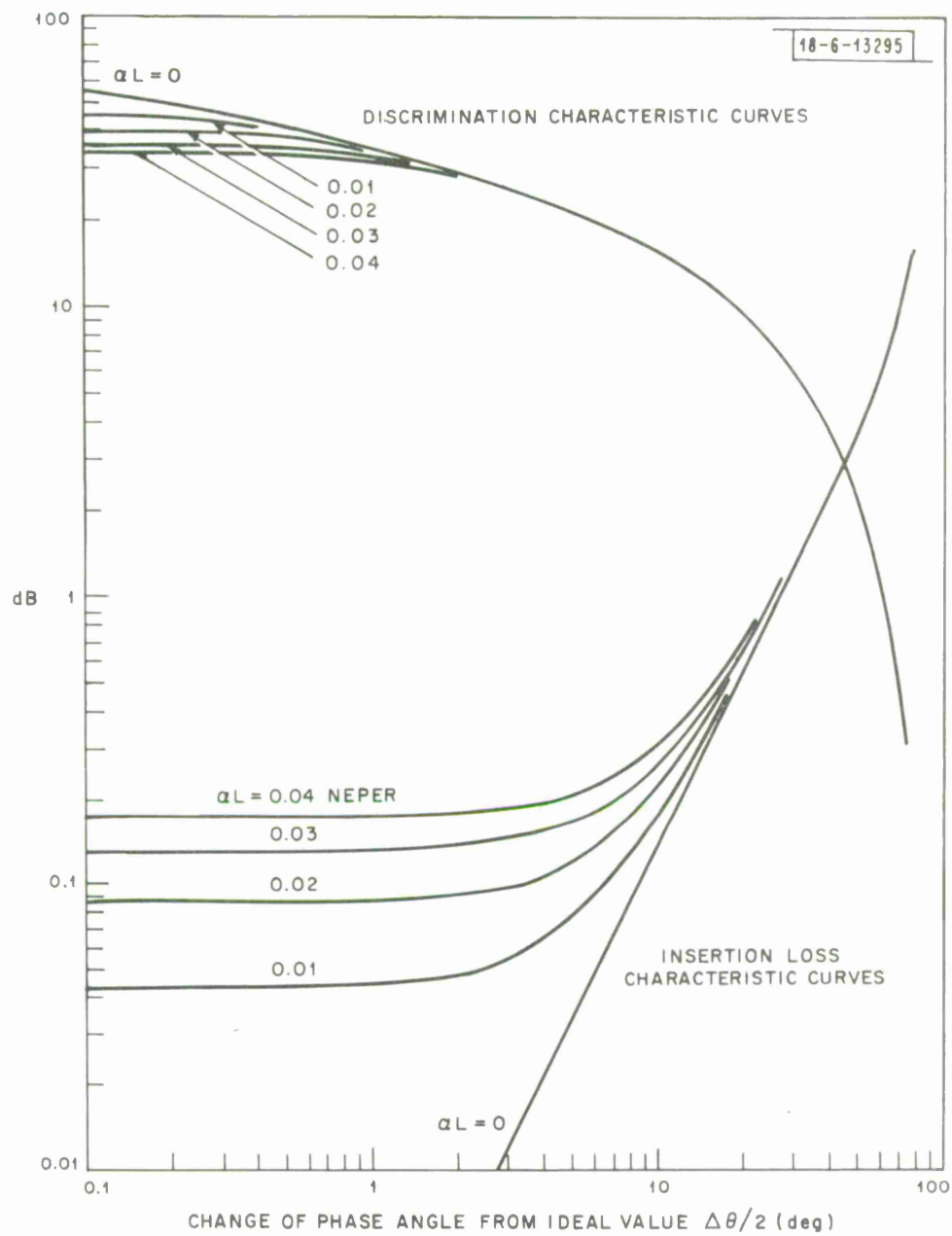
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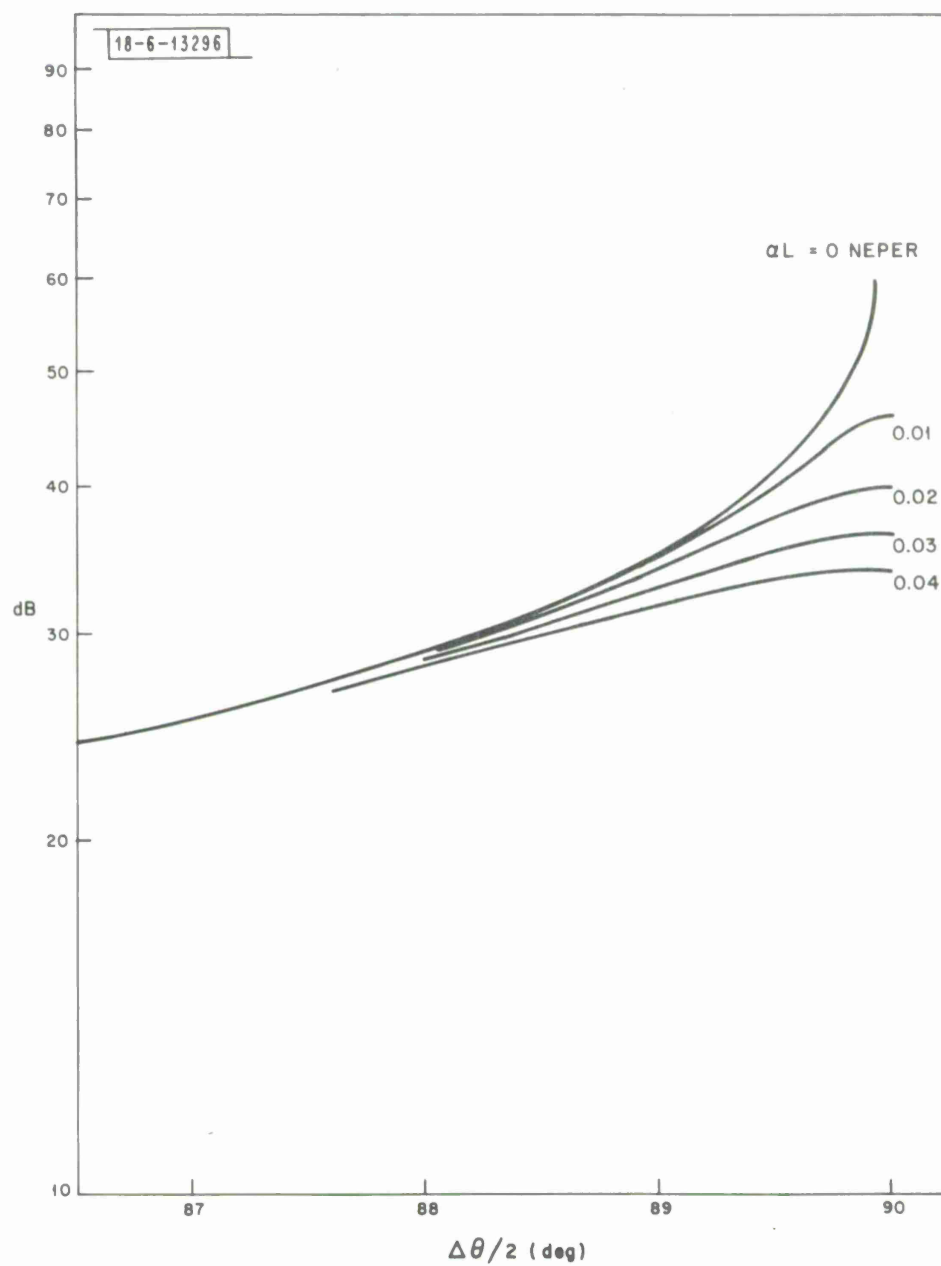Fig. 2(a). Differential line length diplexer general characteristics.

Fig. 2(b).   Differential line length diplexer discrimination characteristics.

$$\lambda_c = \frac{c}{f_2} \left[\frac{\left(\frac{2m}{2n-1}\right)^2 - 1}{\left(\frac{2m}{2n-1}\right)^2 - \left(\frac{f_1}{f_2}\right)^2} \right]^{1/2} \qquad . \tag{5}$$

For a rectangular waveguide operating in the dominant mode, the appropriate guide width is given by

$$a = \frac{c}{2f_2} \left[\frac{\left(\frac{2m}{2n-1}\right)^2 - 1}{\left(\frac{2m}{2n-1}\right)^2 - \left(\frac{f_1}{f_2}\right)^2} \right]^{1/2} \tag{6}$$

where c is the speed of light. From Eq. (6), it is clear that when f_1 and f_2 are close together, the operation will be close to cut-off unless $2m/(2n-1)$ is close to unity. Under such conditions, m and n are large compared to unity and the bandwidth of the device will be narrow as can be seen from the analysis of the next section.

Note that the differential length and the hybrid junctions can be made of different waveguide size, but identical transitions may be used in the two side arms to cancel out any residual phase shift due to the transition.

III. BANDWIDTH AND TOLERANCE ANALYSIS OF THE DIPLEXER

The diplexer performance characteristics across a given frequency band and over certain dimensional and/or thermal expansion tolerances can be

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directly calculated from Eqs. (1) and (2). Since the combination of various parameters are many and the compilation of computer results is tedious, an analytical approach is sought here. Taking the differential of Eq. (4), we have for the dominant mode

$$\frac{\Delta\lambda_g}{\lambda g} \approx \frac{\lambda g^2}{\lambda_o^2} \frac{\Delta\lambda_o}{\lambda_o} - \frac{\lambda g^2}{4a^2} \frac{\Delta a}{a} = \left(1 + \frac{\lambda g^2}{4a^2}\right)\frac{\Delta f}{f} - \frac{\lambda g^2}{4a^2} \frac{\Delta a}{a} \tag{7}$$

where the relation $\frac{\Delta\lambda_o}{\lambda_o} = -\frac{\Delta f}{f}$ has been used. The first-order phase angle perturbation is given by

$$\theta\left(1 + \frac{\Delta\theta}{\theta}\right) \approx \frac{360L\left(1 + \frac{\Delta L}{L}\right)}{\lambda g\left(1 + \frac{\Delta\lambda g}{\lambda g}\right)}$$

$$\approx \frac{360L}{\lambda g}\left[1 + \frac{\Delta L}{L} + \frac{\lambda g^2}{4a^2}\frac{\Delta a}{a} + \left(1 + \frac{\lambda g^2}{4a^2}\right)\frac{\Delta f}{f}\right]. \tag{8}$$

For fixed frequency $\Delta f = 0$, if $\frac{\Delta L}{L} = \frac{-\lambda g^2}{4a^2}\frac{\Delta a}{a}$ then $\Delta\theta = 0$. Thus, differential length deviation can be compensated by a negative waveguide width change. This relation provides a guide line for diplexer tuning. With appropriate waveguide width and differential length, $\Delta a = \Delta L = 0$, we have

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$$\Delta\theta = \frac{360L}{\lambda g}\left(1 + \frac{\lambda g^2}{4a^2}\right)\frac{\Delta f}{f} \quad . \tag{9}$$

The deterioration of the diplexer performance is determined by the maximum $\Delta\theta$ as shown in Fig. 2. For a given $\Delta\theta$, the fractional bandwidth, $\frac{2\Delta f}{f}$, is inversely proportional to $\frac{L}{\lambda g}$ and $1 + \frac{\lambda g^2}{4a^2}$. Thus, for broad bandwidth the differential length in terms of wavelength, or m and n, should be as small as possible, and the frequency of operation should be significantly higher than the cut-off frequency. For comparable bandwidth at the two design frequencies, $L/\lambda g_1$ and $L/\lambda g_2$ should be close and m = n or m = n = 1 are generally used. This choice also guarantees that no additional pass band or rejection band can exist between f_1 and f_2. A similar argument can be employed to determine the effect of waveguide width deviations.

For waveguide thermal expansion, the waveguide dimensions change with temperature linearly according to

$$\frac{\Delta L}{L} = \frac{\Delta a}{a} = \alpha_t \Delta T \tag{10}$$

where α_t is the thermal expansion coefficient and ΔT is the temperature change in °C. The thermal expansion coefficients and relative resistivity for common good conductors are given in Table I.

TABLE I

Conductors	$\alpha_t \times 10^{+6}$ per °C	Relative Resistivity*
Silver	18. 8	0. 95
Copper	16. 1	1. 00*
Gold	14. 3	1. 42
Aluminum	28. 7	1. 64

Putting Eq. (10) into Eq. (8), for maximum $\Delta\theta$ deviation we have

$$\Delta\theta_{max} \approx \frac{360L}{\lambda g} \left(1 + \frac{\lambda g^2}{4a^2}\right)\left(\alpha_t |\Delta T| + \frac{|\Delta f|}{f}\right) \quad . \tag{11}$$

Here we see that temperature induced dimensional changes compete directly with frequency deviation from the center frequency. For common materials, the thermal expansion coefficients are isotropic; consequently, thermal effects upon phase changes are not cancelled by corresponding wave-guide length and width variations.

When both dimensional tolerance and thermal effects exist, we can combine Eqs. (8) and (11) and get

$$\Delta\theta_{max} \approx \frac{360L}{\lambda g} \left[\frac{|\Delta L|}{L} + \frac{\lambda g^2}{4a^2} \frac{|\Delta a|}{a} + \left(1 + \frac{\lambda g^2}{4a^2}\right)\left(\alpha_t |\Delta T| + \frac{|\Delta f|}{f}\right)\right] \quad . \tag{12}$$

Equation (12) is the most general equation which gives the relative amplitudes

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of dimensional tolerance, temperature detuning, and bandwidth contributions to phase angle deviation. By setting $\lambda g/2a = 0$, Eq. (12) can be used for TEM lines or oversized waveguides. It is clear that for these cases the bandwidth characteristics are much improved. This equation, when combined with curves of the general diplexer characteristics, as given in Fig. 2, will give the total result of these effects. An example will be given in the following section.

IV. A DIPLEXER DESIGN EXAMPLE

As an example, an EHF diplexer with $f_1 = 54.5$ GHz and $f_2 = 52.5$ GHz, is given below. For m = n through Eq. (6), the ideal waveguide widths are given in Table II for different n.

TABLE II

n	a(mm)	Remarks	
5	3.491		
6	3.712	WR-15	a = 3.759 mm
7	3.982		
8	4.322		
9	4.766	WR-19	a = 4.775 mm
10	5.383	WR-22	a = 5.690 mm

For n < 5 the operation will be too close to cutoff; and the ohmic loss, band-width, and tolerance problems will be severe as can be seen from the analysis

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of the previous section. For $n > 10$, the higher order modes can propagate. But for $n = 6$ and 9 the ideal waveguide sizes are very close to WR-15 and WR-19 standard waveguides. We will choose to use WR-19 waveguide. With $n = 9$, the differential length is found to be 2.384 inches.

The waveguide dissipation loss in the EHF range is significant. For WR-19 waveguide the theoretical calculated attenuation is about 0.4 dB per foot, but the experimental measured value is about 0.8 dB per foot. With the above given differential length, we have $\alpha L \approx 0.02$ neper. The insertion loss and discrimination characteristics are calculated through Eqs. (1) and (2) with $\alpha L = 0$ and 0.02 neper. These results are plotted in Fig. 3. It is seen that the reduction of discrimination is more localized around the center frequencies than the corresponding increase of insertion loss. Ideally, for an isolation of 22 dB and an insertion loss of 0.11 dB, a bandwidth of 200 MHz (0.37 percent) can be obtained.

All the performance characteristic deviations due to dimensional variations can be calculated directly from Eq. (8). Qualitatively, to get the same performance, an increase of Δf and/or Δa corresponds to a decrease of Δf, and vice versa. Around $f_2 = 52.5$, Eq. (8) yields $\Delta f = 14.2$ MHz per mil of change of L and $\Delta f = 99.6$ MHz per mil of change of a. More information can be obtained directly from Eq. (12) and Fig. 2. For example, the worst case bandwidth of the EHF diplexer defined by a maximum insertion loss of 0.2 dB can be calculated as follows. From Fig. 2, for an insertion loss of 0.2 dB, we
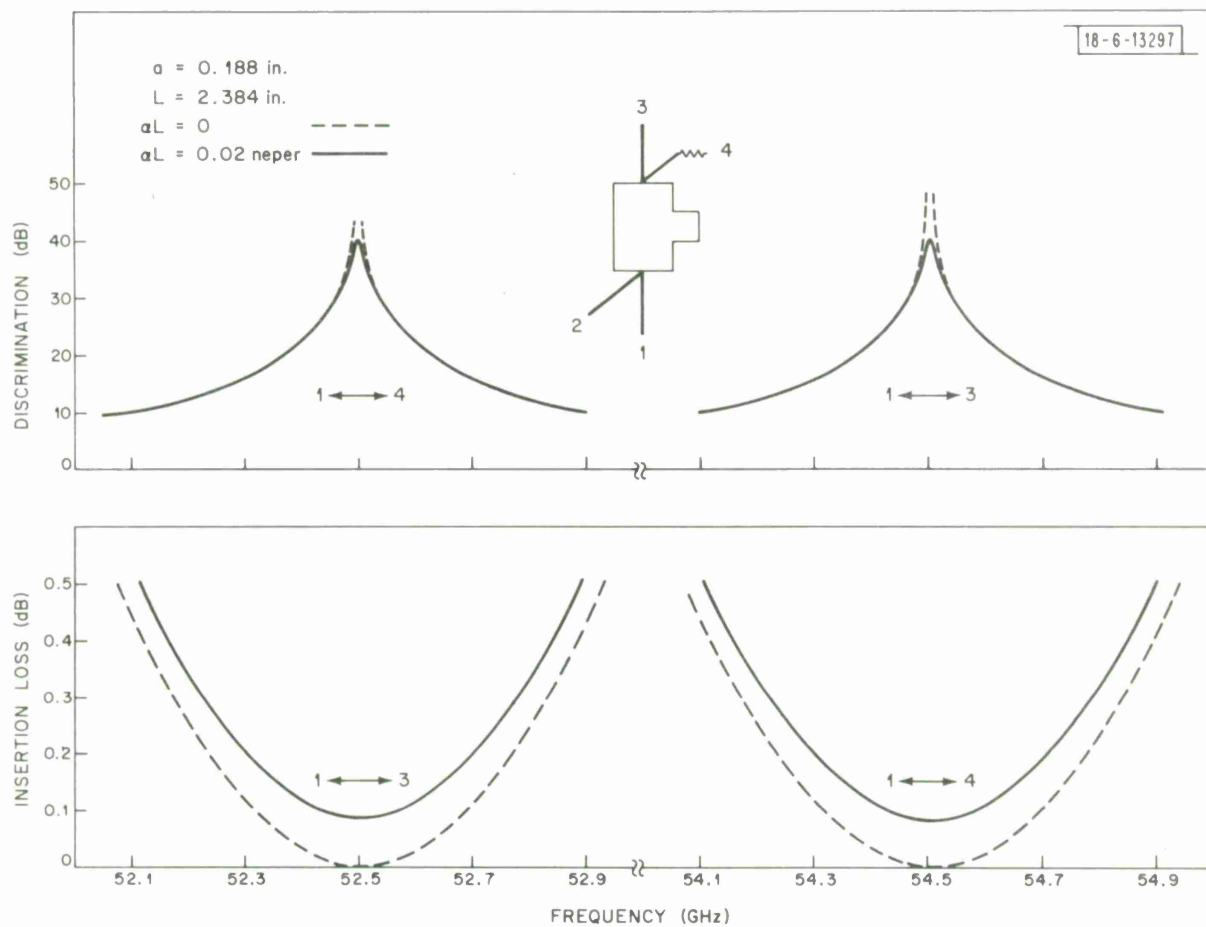
Fig. 3. An EHF differential line length diplexer characteristics.

find $\Delta\theta_{max} = 17.2°$. If the diplexer is made of silver, we have $\alpha = 0.95 \times 10^{-6}$ per °C. With a length tolerance of ± 2 mils and waveguide width tolerance of ± 1 mil and temperature variation of ± 50 °C, we find the usable bandwidth to be 66 MHz (≈ 0.1 percent). If this narrow bandwidth is not acceptable, the tolerance values or temperature variation must be changed. With ± 10 °C, the usable bandwidth is increased to 83 MHz (0.16 percent).

The diplexer total dissipation loss can be calculated easily by adding the straight waveguide loss due to L_1 and the hybrid losses. For $L_1 \approx 1$ inch, the extra loss due to L_1 is 0.07 dB. Commercially available hybrids have a specification of about 0.5 dB for insertion loss. Thus, the total loss may be 1.27 dB. It is seen that the total loss is dominated by the hybrids. To reduce the total loss, improved design for the hybrid is needed.

V.   THE LONG-STUB FILTER

The long-stub filter consists of a section of short-circuited waveguide attached to the main waveguide to form a E-plane or H-plane tee. Figure 4 shows the former configuration of a long-stub filter and its approximate equivalent circuit. Neglecting junction discontinuity reactance and dissipation loss at the short, the series impedance Z_s is given by

$$Z_s = Z_{os} \tanh(\alpha + j\beta) L_s \tag{13}$$

where Z_{os} is the characteristic impedance of the stub waveguide, L_s is the length of the short-circuited stub, α and β are the attenuation and phase constants of the stub waveguide and $j = \sqrt{-1}$.
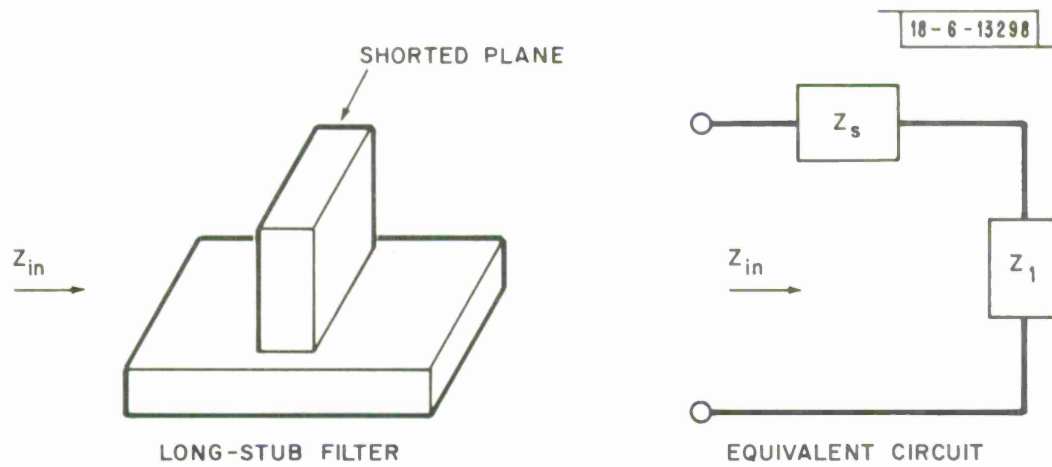
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SHORTED PLANE

Z_{in}

LONG-STUB FILTER

18 - 6 - 13298

Z_s

Z_{in}

Z_1

EQUIVALENT CIRCUIT

Fig. 4. Configuration and equivalent circuit of a long-stub filter.

The VSWR and insertion loss L_i are given by

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{14}$$

$$L_i = 20 \log |1 - \Gamma| \tag{15}$$

where Γ is the voltage reflection coefficient given by

$$\Gamma = \frac{Z_{os} \tanh (\alpha + j\beta) L_s}{2Z_o + Z_{os} \tanh (\alpha + j\beta) L_s} \quad . \tag{16}$$

The general VSWR and insertion loss characteristics in the pass and rejection bands can also be calculated in terms of phase angle change, $\theta = \beta L_s$, with αL_s as parameter. For $Z_o = Z_{os}$ the results are given in Fig. 5. It is seen that the VSWR in the pass band deteriorates slowly as the phase angle deviates from zero (VSWR $\approx$ 1.2 for $\theta = 10°$), but the isolation at the rejection band drops rapidly as the phase angle deviates from $90°$. ($L_i \approx 30$ dB for $\theta = 89.1°$ and $L_i \approx 20$ dB for $0 = 87.1°$, when dissipation loss is negligible.)

By adjusting the length of the short-circuited section of waveguide, it can be made to present a high mismatch at the reject frequency band and a good match at the pass-frequency band. In particular, if the length is chosen such that

$$L_s = 2m \frac{\lambda_{gp}}{4} = (2n-1) \frac{\lambda_{gr}}{4} \tag{17}$$

where L_s is the length of the stub, m and n are integers, and λ_{gr}, λ_{gp} are
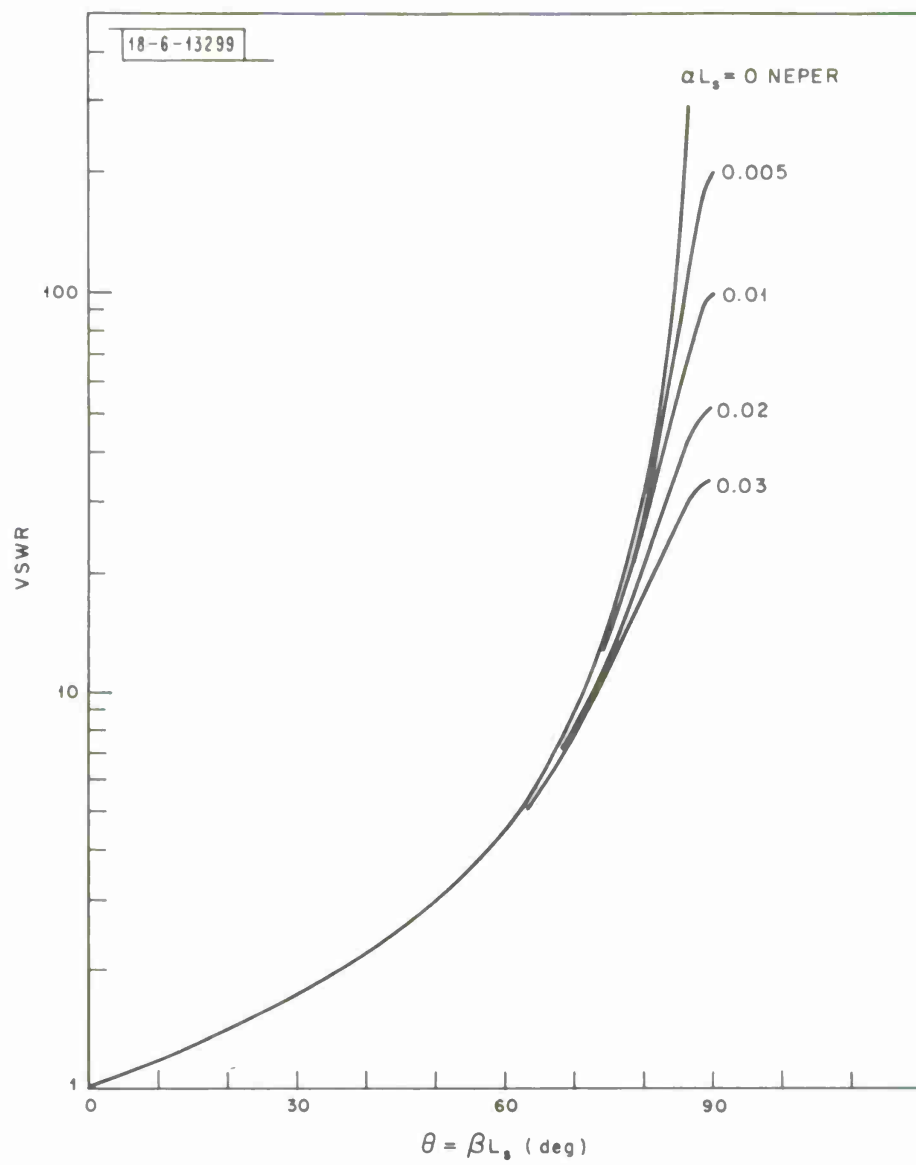
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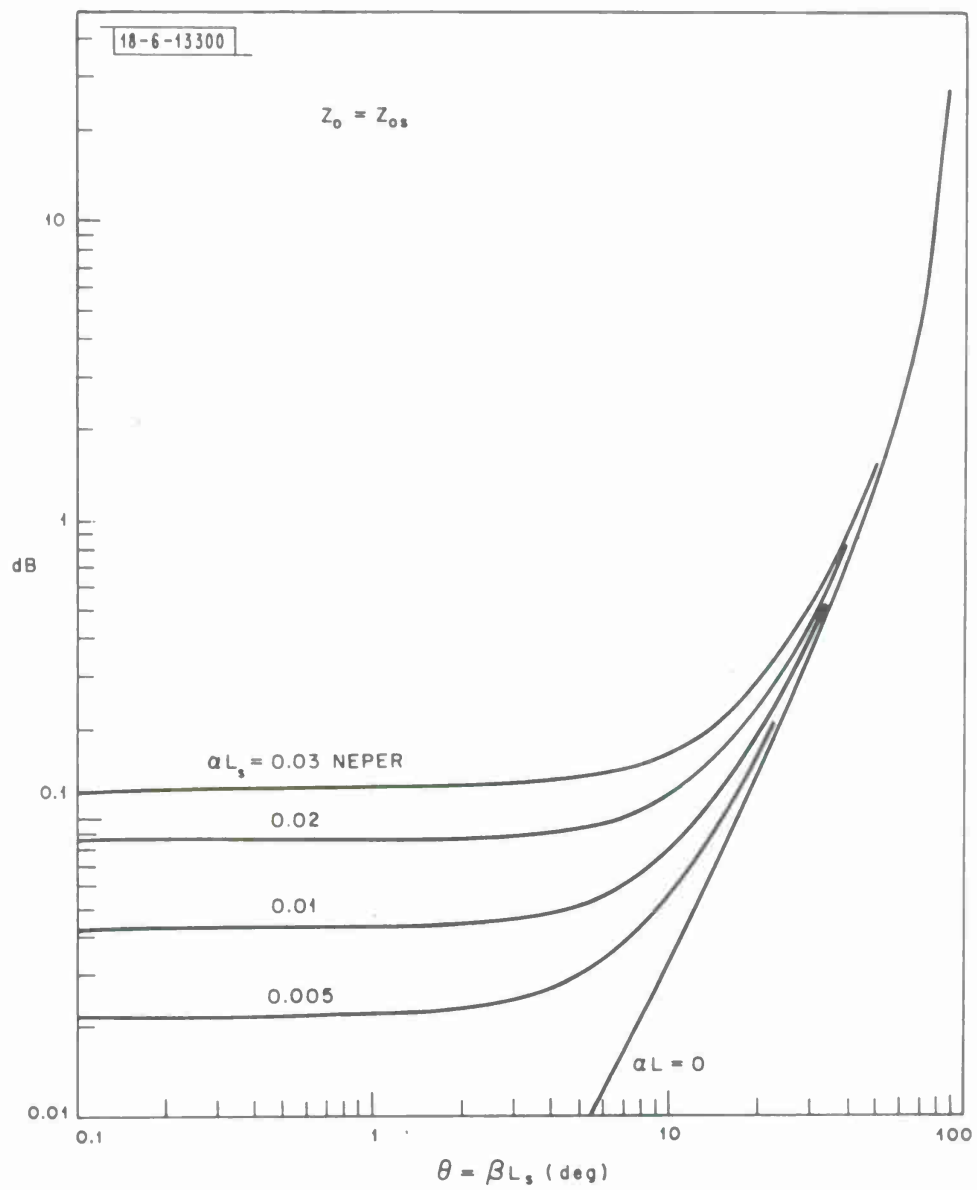Fig. 5(a). Long-stub filter VSWR characteristics.

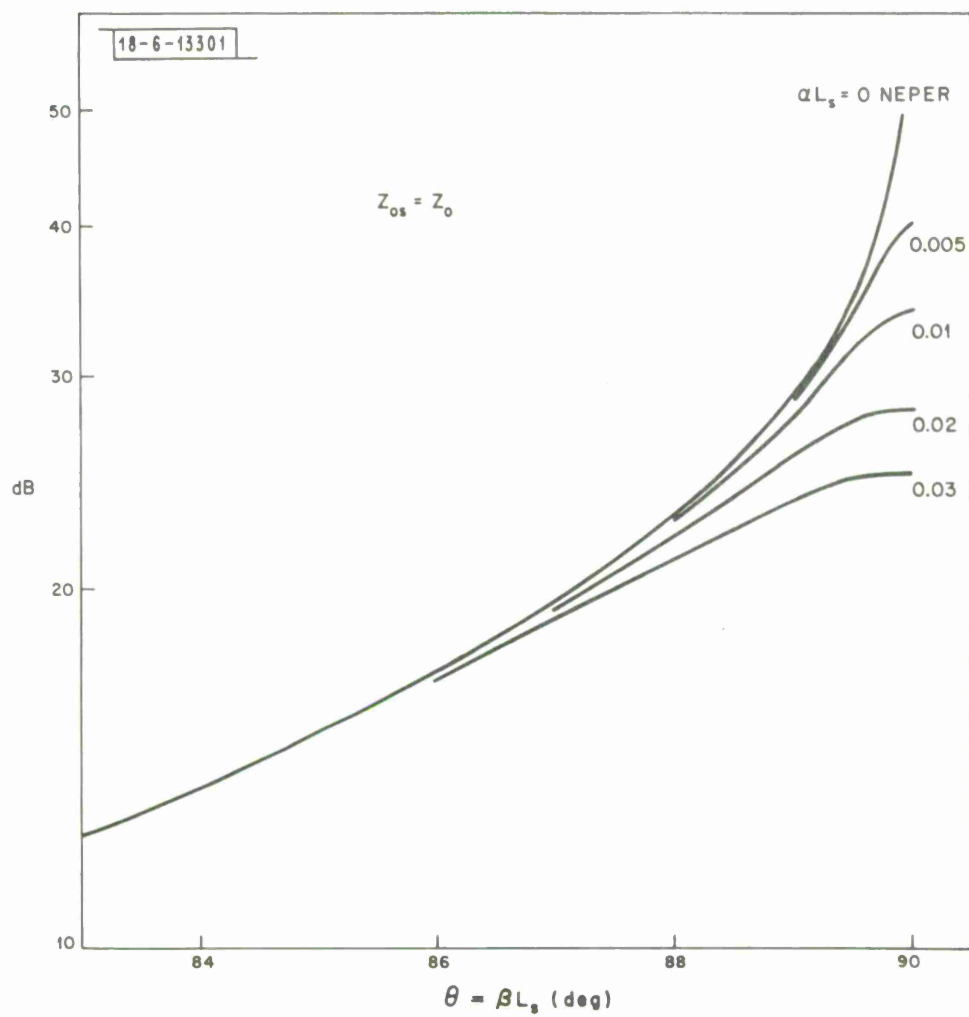Fig. 5(b).   Long-stub filter insertion loss characteristics.

Fig. 5(c).   Long-stub filter rejection characteristics.

the guided wavelengths at the center frequencies of the rejection and pass band. For ideal operation at the two-center frequencies, the ideal waveguide size is given by Eq. (6) with the subscripts 1, 2 replaced by p and r. Thus, the calculated results given in Table II are also good for a filter passing 54.5 GHz and rejecting 52.5 GHz. For the case of passing 52.5 GHz and rejecting 54.5 GHz, we choose n = m + 1, the results from Eq. (6) is given in Table III.

TABLE III

n	a(mm)	Remarks	
5	3.596		
6	3.840	WR-15	a = 3.759 mm
7	4.142		
8	4.528	WR-19	a = 4.775 mm
9	5.046		
10	5.798	WR-22	a = 5.690 mm

If WR-19 is chosen for the main waveguide, the long-stub filter should be made of WR-19 for rejecting 52.5 GHz and a = 4.528 mm should be chosen for rejecting 54.5 GHz.

In conjunction with Fig. 5, the bandwidth dimensional tolerance and temperature detuning analysis for the filter is given by Eq. (12) with L replaced by L_s. For the long-stub filter, rejecting 52.5 GHz and passing 54.5 GHz, made of standard WR-19 waveguide with length tolerance of ± 2 mils and waveguide width tolerance of ± 1 mil and temperature variation of ± 50°c, the 10 dB

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minimum rejection bandwidth is found to be 125 MHz (0. 24 percent). For

αL_s = 0. 01 neper, the pass-band insertion loss is found to be 0. 08 dB. For

15 dB minimum rejection, the waveguide width tolerance of $\pm$ 1 mil alone will

limit the usable bandwidth to 9. 5 MHz (0. 02 percent). It is seen that the band-

width of a single long-stub filter is much narrower than that of a differential

line length diplexer.

When the minimum rejection of bandwidth requirements are greater than

what can be obtained by a single long-stub filter, multiple elements can be

used. (An increase of Z_{os} in respect to Z_o only increases the isolation slowly.

For example, by doubling Z_{os} the maximum isolation will increase only 6 dB.)

If n elements are spaced at quarter wavelength intervals, the overall attenua-

tion can be increased by n times that of a single element filter.[3] It is inter-

esting to note that the mismatch due to the neglected discontinuity reactances

also tends to cancel in the pass band by the quarter wavelength separation ar-

rangement.

VI. DISCUSSION AND CONCLUSION

From the point of view of tolerance and size, both the differential line

length diplexer and the long-stub filter are well suited at frequencies above

X-band. The general performance characteristics of these devices are

basically determined by trigonometric functions. (Sine, cosine for the di-

plexer and tangent for the filter.) To fully utilize these characteristics, the

appropriate waveguide width and length for perfect performance at two given

center frequencies have been derived. The general characteristics of these

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devices have been calculated in terms of phase angle with dissipation factor as a parameter. The results are presented in the form of "universal" curves. An analysis of bandwidth, dimensional and thermal expansion tolerances for these devices is also given. A single algebraic equation relating to the di-plexer performance and the various parameter first-order deviations is ob-tained. This relation gives guide lines for adjusting final performance and for predicting the combined effects of dimensional, thermal and frequency deviations upon the general performances. Examples of diplexer and long-stub filter designs and analyses are given.

ACKNOWLEDGMENT

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13. ABSTRACT

Formulas for calculating the appropriate waveguide width for perfect performance at two center frequencies have been derived for the differential length diplexer and long-stub filter. The general characteristics of these devices are calculated in terms of phase angle with dissipation loss included. An analysis of bandwidth, dimensional and thermal expansion tolerances for these devices is also given. A single algebraic equation relating the diplexer performance and the various parameter first-order deviations is obtained. This relation gives guide lines for adjusting final performance and for predicting the combined effects of dimensional, thermal and frequency deviations upon the general performances. Examples of diplexer and long-stub filter designs and analyses are given.

14. KEY WORDS

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waveguide

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X-band

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